Phobos control point network, rotation, and shape

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A new independent control point network for Phobos was computed from image data obtained by the SRC (Super Resolution Channel) on board the European Mars Express Mission. The network solution includes 3D coordinates of 665 surface control points and was used to observe the forced libration amplitude of Phobos. Based on the network control points a spherical harmonic function model to degree and order 17 was derived, from which volume, bulk density and moments of inertia were computed. The modeled forced libration amplitude agrees to our observation within the error bands, indicating a homogeneous mass distribution for Phobos. To bring both values into exact agreement with the observations, different mass distribution models were applied. It appears that the amplitude is relatively insensitive to a simple two-layer density model.

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1. Introduction

In 1877, Asaph Hall, an astronomer at the United States Naval Observatory, discovered that Mars is accompanied by two satellites, Phobos and Deimos. The origin of the two has remained uncertain to the present day. While the two satellites may represent ejecta from Mars that reaccreted in planet orbit, their origin as captured asteroid fragments from the main belt cannot be ruled out. Spectral signatures (Bell et al., 1993; Pieters et al., 1999) and the overall density (approx. 1.9 g/cm³) suggest that Phobos (the larger of the two) is similar in composition to carbonaceous chondrites and therefore raises the possibility that Phobos contains a large fraction of volatiles. US and Russian space mission planners have identified Phobos as a target, from where the recovery of extraterrestrial samples may be comparably straightforward (Pieters et al., 1999; Marov et al., 2004).

Clues on the origin of Phobos and Deimos may be obtained from their overall shape, surface morphology, and interior structures. Phobos is orbiting Mars in a near-circular near-equatorial orbit with a mean distance to the center of the planet of 9375 km, i.e., deep in the gravity field of Mars. Inferences on its interior structure may be obtained by studies of Phobos' librational motion, caused by tidal forces interacting with the odd shape of the satellite.

While early data on size, shape, and rotation of Phobos were obtained during the Mariner-9 and Viking missions (Duxbury and Callahan, 1989), we concentrate on the analysis of more recent image data obtained by the SRC (Super Resolution Channel) of the HRSC (High Resolution Stereo Camera) on the European Mars Express spacecraft (MEX).

In this paper, we present a new control point network which includes a solution for the librational motion of Phobos. From the control point data we derive a model for the shape of Phobos and we report new values for total volume and moment of inertia factors. We finally discuss implications for the interior structure of the satellite.

2. Previous shape models

Various previous models exist to describe the shape of Phobos. Limb analyses were used to derive ellipsoidal models (Thomas, 1989). The combination of limb observations with 3D coordinates of control points led to a numerical shape model with a 2×2 degree grid spacing (Simonelli et al., 1993).

Other modeling efforts involved the identification of landmarks and image block adjustments to derive large numbers of control point coordinates. The Turner (1978) shape model involved 260 control point coordinates. Spherical harmonic expansion models were fitted to control point coordinates by Duxbury (1991) to degree and order 8. Approx. 315 craters with depths-to-diameter ratios between 0.1 and 0.2 were added separately to the shape model for added detail (see also Table 1).

In this paper, we compute a network of 3D coordinates for a large number of surface points fitted by spherical harmonic expansion models.
3. Control point network

3.1. Duxbury and Callahan control point network

From July 1976 to October 1978, the two Viking orbiters and their framing cameras obtained global image coverage of Phobos. Image resolutions were in the order of 200 m/pixel and better, showing Phobos under excellent phase angles. Duxbury and Callahan (1989) established one of the first global control point networks consisting of 98 ground control points (GCP). Duxbury (1991) extended the network for a total of 315 points. Control points were represented by craters of various sizes, covering several pixels. Uncertainties of the 3D-coordinates, which referred to the centers of local planes on the crater rims, ranged from ±74 m to ±900 m.

3.2. SRC image data

Mars Express is on an elliptical trajectory and currently the only Mars orbiting spacecraft making regular Phobos flyby maneuvers. As of December 2008, the spacecraft has engaged in 114 Phobos flybys, and the SRC obtained 345 images of Phobos with resolutions ranging between 100 m/pixel and 0.9 m/pixel. The SRC is a 1 k×1 k framing camera with a large focal length of 988.5 mm (Oberst et al., 2008).

During a flyby, the SRC is pointed at a fixed direction in the stellar sky. Hence, multiple coverage of areas, observed under different viewing angles during different flybys, is required to determine 3D-Coordinates of control points. SRC images are covering approx. 84% of the surface of Phobos, approx. 75% in stereo (cf Fig. 1).

Viking Orbiter (VO) framing camera images were used to fill gaps (20%) in the image coverage within the area between 180° and 270° West. Control point measurements in SRC and VO images were combined in the block adjustment to establish a global coverage at high spatial resolution. Likewise, HRSC data were used to fill remaining gaps (estimated 10%) in the shape model, though HRSC control points were not included in the adjustment because of the comparably low HRSC image resolutions.

3.3. Object point determination

Suitable control points were selected and their line/sample coordinates were measured in 53 SRC images and 16 VO images. Pixel resolution range from 5 m/pixel to 48 m/pixel but on average 17 m/pixel for SRC images and from 6 m/pixel to 77 m/pixel (average 17 m/pixel) for VO images. Contrary to the definition of the control points of Duxbury and Callahan (1989), where crater coordinates represented the center of a local plane on the crater rims, control points were defined as the centers of the crater floors. Image resolutions permitted to observe very small surface features — even small features within larger craters. Hence, we estimated that the observed points represent the mean surface with a better approximation.

A total of 665 points were observed 3898 times with a minimum of 2 observations and a maximum of 14 observations, but on average 6 observations per point in both image data sets.

For the block adjustment camera orientation data are transformed into the Phobos body fixed coordinate space. To detect gross errors in the predicted orientation of the cameras, least-squares adjustments were computed for both data sets separately. Orientation data for the SRC was of good quality and could directly be used to determine object point coordinates in the bundle block adjustment. However, normalized residuals indicated larger errors than the preliminary assumed uncertainties for camera orientations. Significantly improved results were computed after an adopted weighing scheme was applied to the camera orientation data. Mean object point accuracies \( \sigma_x, \sigma_y, \sigma_z \) of 27.1 m, 17.2 m, and 19.5 m, respectively, were computed for control points measured in SRC images. An uncertainty of 1 pixel for the image coordinate observations was assumed.

The orientation data of Viking orbiter images are known to suffer from large errors (Zeitler, 1999). Hence, we applied corrections to the orientation of the Viking orbiter cameras prior to a bundle block adjustment by fitting the predicted limb position to the observed position of Phobos in the images. This relates to rotations about two axes of the camera. Very high resolved images do not necessarily depict parts of the limb. Therefore, an overlay was produced, containing the control point positions of the Duxbury (1991) control point network, which was then fitted to the surface features. The stochastic model was again adjusted according to the computed normalized residuals of the orientation data. The least-squares bundle block adjustment of the Viking orbiter data set was very sensitive to variations of the stochastic model. Due to the uncertain VO navigation data, we re-computed the positions and orientations for VO observations during a first least-square bundle block adjustment. The computed camera orientations for SRC and the VO cameras were included in a second bundle block adjustment to compute the object point coordinates of the GCPs. A Baarda gross error detection was applied (Baarda, 1968) to rule out misidentified point observations. After removal of these points, uncertainties of the 3D ground coordinates were reduced marginally for points in SRC images and by a factor of approx. 6 for points in Viking images (cf. Table 2) when computing the combined data set.

3.4. Reference frame

We note that the center of figure was not explicitly observed and introduced as a control point during this analysis. Hence, the resulting 3D-coordinates are not tied to the center of figure of Phobos. Instead the control point reduction relies on the computed position of Phobos from an orbit prediction model. In the course of this analysis we used the latest JPL release of ephemerides of the Martian satellites MAR080 (Jacobson, 2008a,b). All results are related to this orbit prediction model and may differ if other ephemerides are used.
A list of determined ground control point coordinates is given in the Appendix in Table A1. Coordinates and 1σ residuals are given in meter.

4. Spherical harmonic functions

To solve for the coefficients of the harmonic function the 3D-coordinates of the 665 GCPs were converted to spherical coordinates, and spherical harmonic functions were fitted to the points. The radii, \( u(\phi, \lambda) \), of the points observed at longitude, \( \lambda \), in positive East direction and latitude, \( \phi \), are expressed in a series of spherical function terms, where \( A_{nm}, B_{nm} \) are the coefficients to be determined (Eq. (1))

\[
u(\phi, \lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} [A_{nm} \cos m\lambda + B_{nm} \sin m\lambda]P_{nm}(\sin \phi)
\]

with \( 90^\circ \geq \phi \geq -90^\circ \).

The subscripts \( n \) and \( m \) of the coefficients indicate the degree and order, respectively, of the spherical function.

The expansion was developed up to degree and order 17, resulting in a total of 324 coefficients to be determined (see Table A2 for the full set of coefficients). Due to the relative high redundancy of almost 2.5 observations per unknown parameter, the resulting figure is well constrained in areas where many control points were observed. However, especially in the area between 180° and 270° West near the North Pole of Phobos, only few GCPs could be determined from low resolution VO images. Hence, artificial bulges are visible in the expansion model in this area when using the 665 control points.

Fortunately, during the flyby in MEX orbit 5851 on 23 July 2008 with a distance of only 93 km to the surface of Phobos, the HRSC imaged this area to large parts with a resolution of 4 m per pixel in the primary stereo channels. The HRSC image swaths of the MEX orbit 5851 flyby were tied to the SRC/VO control point network by holding the coordinates of one already determined object point fixed while improving the orientation of the different HRSC channels to each other. Thus the orientation was tied to the control network. The quality of this tie was determined by comparing coordinates of other GCPs computed from the HRSC image adjustment and the control network. Coordinates were statistically equal as no difference between the coordinate sets of more than \( \pm 15 \) m was found.

A total of 90 GCPs from the HRSC stereo analysis was introduced in the expansion model analysis, significantly constraining the spherical harmonic function model. Though a higher degree and order model resulted in further reduction of the residuals (Fig. 2), the large spacing of control points in some areas caused again artifacts in the model. The spherical harmonic function model represents the shape of Phobos with good detail (Fig. 3).

Fig. 1. Equidistant map of current surface coverage by SRC images of Phobos (updated: 26th Aug. 2008).

Fig. 2. Residuals of the shape model to the GCPs after fitting expansion models of various degree and order.
5. Physical and dynamical parameter

5.1. Geometric properties

We used the determined spherical harmonic function model up to degree and order 17 for further analysis. The analytic expression was used to compute radii of 64,800 points on the surface of the model in a 1° spacing. In a least-square fit the radii of the best fitting triaxial ellipsoid were computed to be $a = 13.00$ km, $b = 11.39$ km and $c = 9.07$ km. We note that these radii are the best fit in radial distance from the COF between points on the ellipsoid and the surface model. The determined radii are smaller than the previously published values for Phobos (Seidelmann et al., 2002). The mean difference between the radii is $\pm 250$ m.

The degree and order one coefficients of the spherical expansion model revealed that the center of figure is located at $x = -375.9$ m, $y = -341.8$ m, $z = 206.3$ m with respect to the coordinate frame the control points were observed in (Duxbury, 1989). The coordinate frame of the ground control points is additionally rotated about the Z-axes, Y-axes and X-axes by $0.69^\circ$, $-0.73^\circ$ and $-0.59^\circ$, respectively, with respect to the principle axes of inertia. Since Phobos’ center of figure was not explicitly observed during the control network analysis, it is more likely that the translation from coordinate frame origin to COF of the shape model represents the remaining differences of Phobos’ true position and its predicted position (Willner et al., 2008) or an effect resulting from the non-uniform distribution of control points.

5.2. Volume and bulk density

Direct integration of the derived analytical expression for the shape of Phobos to compute the volume or moments of inertia proved to be expensive in terms of time and computational costs. Hence, we divided the modeled shape into discrete small sized cubes. The volume was computed by integration and found to be 5689.3 km$^3$. This value agrees well with results of Duxbury (1991) while the derived uncertainty is significantly lower in comparison to previous estimates (cf. Table 3).

Fig. 3. Panels (a) and (b) show Phobos’ leading and trailing side, respectively, as modeled with the degree and order 17 spherical harmonic function model. 665 plus 90 additional introduced points from the HRSC observation during orbit 5851 were used to compute the coefficients of the expansion model. An example of one SRC observation is given in panel (c) which is compared by with the model in panel (d).
The residuals of the GCPs, indicate an amplitude of 1.2° for the forced orientation of Phobos systematically. Subsequent studies of the orientation of Phobos.

The body of Mars causing a superimposed sinusoidal oscillation on the rotation of Phobos. We concentrated on the observation of the forced libration amplitude through the control point network assuming a 1:1 ratio of Phobos. We determined a new global control point network on the basis of SRC images. The control network was used to model Phobos' shape and rotation as well as to observe the forced libration amplitude. The shape model is in good agreement with previous models (Duxbury, 1991; Simonelli et al., 1993) but shows significantly more detail. Large craters, such as Stickney, Hall, and Druinio, can, in contrast to previous models, be clearly identified.

Table 3
Comparison of re-estimated property values of Phobos with previous determined values.

<table>
<thead>
<tr>
<th>Property</th>
<th>Previous estimation</th>
<th>Value</th>
<th>Re-estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radii</td>
<td>Duxbury (1989)</td>
<td>12.61 × 11.33 × 8.60 km</td>
<td>13.00 × 11.39 × 9.07 km</td>
</tr>
<tr>
<td>Thomas (1989)</td>
<td></td>
<td>13.4 × 11.2 × 9.2 km</td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>Duxbury (1991)</td>
<td>5680 ± 250 km³</td>
<td>5689.8 ± 60 km³</td>
</tr>
<tr>
<td>Moments of inertia (normalized)</td>
<td>A, B, C</td>
<td>0.3362, 0.3871, 0.4773</td>
<td>0.3615, 0.4265, 0.5024</td>
</tr>
<tr>
<td>Forced libration amplitude</td>
<td>Observed</td>
<td>0.8 ± 0.3°</td>
<td>1.2° ± 0.15°</td>
</tr>
<tr>
<td>Modeled</td>
<td>Duxbury (1991)</td>
<td>0.9°</td>
<td>1.1°</td>
</tr>
</tbody>
</table>

The observed volume of 5689.8 ± 60 km³ was computed, which is also in correspondence to the previous determined values, (Duxbury, 1991) and (Turner, 1978), of 5680 ± 250 km³ and 5620 km³ (no error estimate given), respectively. Depending on the mass estimate used a bulk density of 1.867 g/cm³ to 1.885 g/cm³ was computed. The low densities attest to the high porosity of Phobos (see Table 4).

Table 4
Computed bulk density depending on current mass estimates.

<table>
<thead>
<tr>
<th>Reference</th>
<th>CM (km³/s²)</th>
<th>Bulk density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Konopliv et al. (2006)</td>
<td>0.0007158</td>
<td>1.885</td>
</tr>
<tr>
<td>Jacobson (2008a,b)</td>
<td>0.0007112</td>
<td>1.873</td>
</tr>
<tr>
<td>Andert et al. (2008)</td>
<td>0.000709</td>
<td>1.867</td>
</tr>
</tbody>
</table>

The observed volume of 5689.8 km³ was used.

6. Discussion

We determined a new global control point network on the basis of SRC images. The control network was used to model Phobos' shape and rotation as well as to observe the forced libration amplitude. The shape model is in good agreement with previous models (Duxbury, 1991; Simonelli et al., 1993) but shows significantly more detail. Large craters, such as Stickney, Hall, and Druinio, can, in contrast to previous models, be clearly identified.

While the overall shape models are in good agreement with our expansion model, the radii of the best fit ellipsoid appear somewhat smaller than values recommended by the IAU (Seidelmann et al., 2007). However, determined radii of 13.00 km × 11.39 km × 9.07 km (Table 3) agree to values obtained by Borderies and Yoder (1990) and by Duxbury (1989).

The observed amplitude of the forced libration of 1.2° with an estimated error of ±0.15° is within error bands to the previously reported amplitude of 0.80 ± 0.3° (Duxbury, 1989) and 0.78° (Seidelmann et al., 2005). Chao and Rubincam (1989) attribute such amplitude to an oscillation of approximately 300 m at the Phobos equator (compared with the average SRC image pixel size of 17 m/pixel).

With the equations

\[ \gamma = \frac{B - A}{C} \theta_n = \frac{2e}{1 - e^2} \] (2)

the moments of inertia of a body can be put into relation with the forced libration too. For the forced libration amplitude, \( \theta_n \), we computed a value of 1.1° using Eq. (2), where \( e \) equals the orbital eccentricity of Phobos and \( A \leq B \leq C \) are the moments of inertia along the principle axis of inertia.

The shape model and the observed forced librations are consistent within the error limits. The observed amplitude differs slightly from the amplitude of 0.8 ± 0.3° observed by Duxbury and Callahan (1989) as well as by Duxbury (1991) but is in good agreement with other estimations such as Borderies and Yoder (1990).
discrepancies, among other reasons, to differences in the coordinate settings which subsequently results in different shape models.

Forced libration amplitudes derived from observations and the shape model and the assumption of a homogeneous mass distribution are in very good agreement.

Nevertheless, to bring predicted and observed forced libration amplitudes to exact agreement we modeled a two layered density distribution and computed the moments of inertia. We assume a global regolith layer of constant thickness with a density of 1.6 g/cm$^3$ (Busch et al., 2007). In order to match the observed amplitude of 1.2°, a 5000 m thick layer with a density of 1.6 g/cm$^3$ would be required, implying a remaining "core" with a bulk density of 2.8 g/cm$^3$. We note that the forced libration amplitude (which strongly depends on the difference between the equatorial moment factors $A$ and $B$, see Eq. (2)) is very insensitive to variations in layer thickness of the model. Instead, mass anomalies located on the Phobos equator (e.g. a mass deficit associated with the Stickney crater) would more strongly affect the librational motion of Phobos.

Dramatic improvements in our knowledge of Phobos orbit, shape, rotation, and interior structure are expected, when image and tracking data from spacecraft, captured in Phobos orbit become available. Such rotation, and interior structure are expected, when image and tracking the librational motion of Phobos. 


Appendix A. Supplementary data


References


